Compact Representation of Spatial Modes of Phase-Sensitive Image Amplifier

Muthiah Annamalai,[1] Nikolai Stelmakh,[1] Michael Vasilyev,[1] and Prem Kumar[2]
[1]*Department of Electrical Engineering, University of Texas at Arlington, Arlington, TX 76019*
E-mail: vasilyev@uta.edu
[2]*Center for Photonic Communication and Computing, EECS Department, Northwestern University, 2145 Sheridan Rd., Evanston, IL 60208*

Abstract: We compute eigenmodes of spatially-broadband optical parametric amplifier with elliptical Gaussian pump and find compact representation of well-amplified modes by the space of the first few Laguerre- or Hermite-Gaussian modes of appropriate waist.

OCIS codes: (190.4970) Parametric oscillators and amplifiers; (270.6570) Squeezed states; (999.9999) Optical image amplification.

Traveling-wave phase-sensitive optical parametric amplifiers (PSAs) can be used for noiseless image amplification [1,2] and multimode squeezed-light generation owing to their wide spatial bandwidth. However, the analysis of the modal structure of these devices is very difficult. This is because their traveling-wave nature requires the use of a tightly focused pump beam to achieve any noticeable gain. The resulting spatially varying PSA gain, together with limited spatial bandwidth, couples and mixes up the modes representing the informational content of the image. Similar spatial mode-mixing effects also make it difficult to detect the lowest-noise mode of a traveling-wave quantum squeezer, because the matched homodyne detector requires exact knowledge of this mode's spatial profile.

We have recently developed a procedure for finding the orthogonal set of independently squeezed (or amplified) modes of a traveling-wave PSA [3]. It is based on solving the parametric amplifier equation via a 2D Hermite-Gaussian (HG) expansion over TEM_{mn} modes, originally developed for cavities [4] and for 1D traveling-wave PSAs [5]. Unlike the previous work, our method can a) handle amplification by an elliptical TEM_{00} pump with potentially different $1/e$ intensity radii a_{0px} and a_{0py} in x- and y-dimensions, respectively, and b) find the eigenmodes of the spatially-broadband PSA once the Greens' function of the PSA is computed [6]. For convenience, we expanded the signal over TEM_{mn} HG modes with the same Raleigh ranges $z_{Rx}=k_p a_{0px}^2$ and $z_{Ry}=k_p a_{0py}^2$ as the pump (i.e. with $2^{1/2}\times$ larger waist) and found the PSA Green's function by solving the resulting system of coupled-HG-mode equations.

Although our original approach [3] has provided a simple and efficient way of finding the PSA Green's function, it also had a serious drawback: the choice of the signal's HG expansion basis required the use of a large number (proportional to the square of pump-beam area) of HG modes, which both demanded large computing resources and complicated the interpretation of the results. In this paper, we report a better HG basis (for elliptical pump case; or Laguerre-Gaussian, or LG, basis for circular pump case), which provides far more compact representation of the PSA modes. This compact basis differs from the original HG basis by the choice of the beam waist: instead of the waist $2^{1/2}\times$ larger than the pump waist, it has a waist that is roughly equal to the geometric average of the pump waist and the inverse spatial bandwidth of the PSA. The resulting requirement on the number of HG or LG modes needed for solving the PSA propagation equations grows linearly rather than quadratically with pump-beam area, which drastically reduces the memory needs and the computation time. Even more importantly, in the new basis the PSA eigenmodes can be approximated by just a handful of HG or LG modes, making it easy to interpret the findings and implement them experimentally.

Figure 1a shows the computed gains of most-amplified eigenmodes for various pump spot sizes. We have assumed a PPKTP crystal with 2-cm length, with pump power adjusted to produce similar gains for eigenmode #0 in all cases. The HG expansion coefficients $|A_{mn}|^2$ for three modes of the most elliptical of these pump beams are shown in the original basis (Fig. 1b) and the compact basis (Fig. 1d), along with their spatial profiles (Fig. 1c). The eigenmodes #0 and #6 have overlap of 98.4% with TEM_{00} and 94.5% with TEM_{60}, respectively. The modes within –3 dB from the maximum gain (gain of mode #0) are well represented by either one or, at most, two HG modes, whereas the eigenmodes outside of the –3-dB bandwidth require a handful of HG modes for representation. The overlap of mode #0 with TEM_{00} can be improved to 99% by tweaking (to $113\times38.4\ \mu m^2$) the waist of the expansion basis for best overlap with mode #0, at the expense of slightly worse overlap with higher-order eigenmodes.

The case of a circular pump is illustrated in Fig. 2, showing the spatial profiles of the first 14 modes (Fig. 2a; some of the modes are double-degenerate with respect to azimuthal rotation), as well as their representations in the original HG basis (Fig. 2b) and in the compact LG basis (Fig. 2c). It is easy to see that, unlike the original basis, the compact basis enables representation of each of the 14 most prominent eigenmodes by a superposition of 4 or fewer

LG modes. The signal spot size was chosen for the maximum 96.5% overlap of mode #5 with LG_{10}, also yielding 98% overlap of mode #0 with LG_{00}. The overlap of mode #0 with LG_{00} can be improved to 99.4% by slightly adjusting the waist of the expansion basis (to 62×62 μm^2), which reduces the mode #5 overlap with LG_{10} to 89.9%.

While there is a big difference in the representation between the original basis and the new compact basis, moderate changes in the basis waist size around the optimum do not reduce the compactness. This means that the approximately optimal waist size can be computed as a geometric average of the pump waist and the inverse spatial bandwidth before solving the PSA equation. The equation is then expanded over the optimum basis and the coupled-mode equations are efficiently solved owing to drastically reduced number of the required expansion modes.

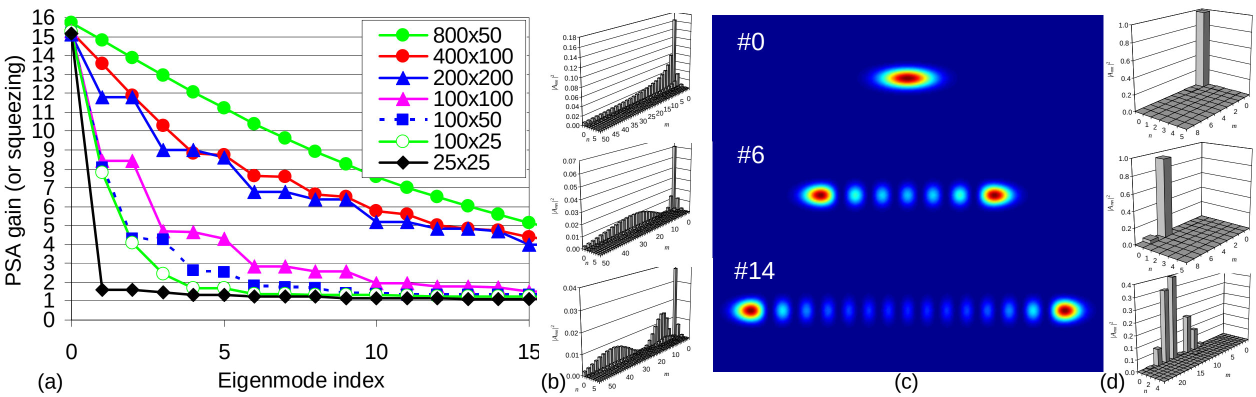

Fig 1. (a) Eigenvalue (gain and squeezing) spectra for various pump spot sizes. (b)–(d) Modes #0, 6, and 14 for the case of an elliptical 800×50 μm^2 pump waist in either (c) *xy*-representation or HG representation with (b) signal waist $2^{1/2}$× larger than pump waist or (d) signal waist of 128×41 μm^2 chosen for maximum (98%) overlap with mode #4.

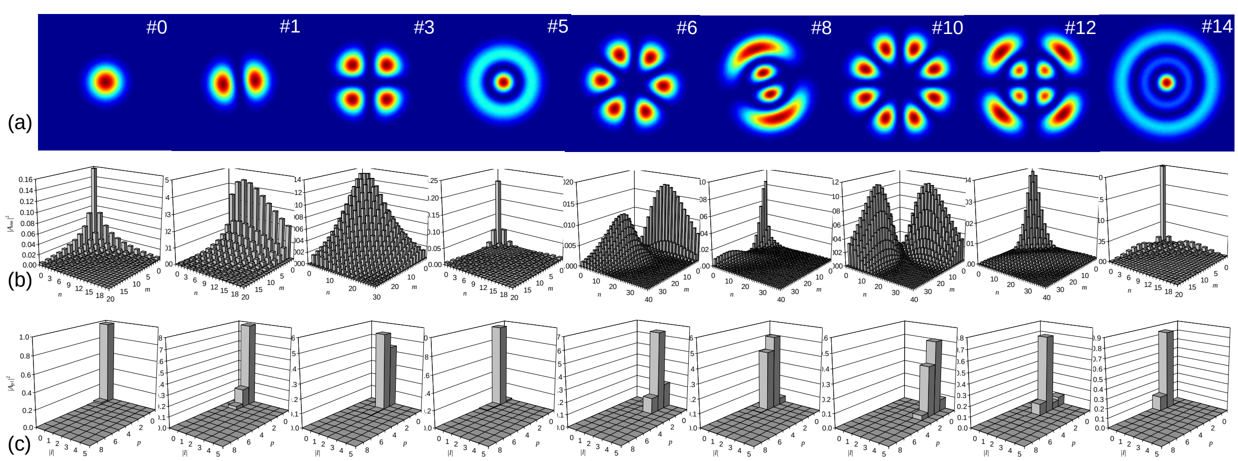

Fig. 2. Most-amplified eigenmodes for the case of a circular 200×200 μm^2 pump waist in (a) *xy*-representation, (b) HG representation with signal waist $2^{1/2}$× larger than pump waist, and (c) compact LG representation with signal waist of 70×70 μm^2 chosen for maximum (96.5%) overlap with mode #5.

To summarize, we have found the eigenmodes of a spatially-broadband PSA and expressed them as superpositions of a very small number of HG or LG modes. This procedure is important for both amplification (boosting image power before detection) and squeezing (suppressing the quantum noise in many spatial modes) applications of the PSA, where it would help, respectively, in minimizing the amplified image distortions and in providing matched local oscillator for maximum squeezing detection.

This work was supported by the DARPA Quantum Sensor Program under AFRL Contract FA8750-09-C-0194.

References

[1] S.-K. Choi, M. Vasilyev, and P. Kumar, Phys. Rev. Lett. **83**, 1938 (1999).
[2] E. Lantz and F. Devaux, J. Sel. Top. Quant. Electron. **14**, 635 (2008).
[3] M. Annamalai, N. Stelmakh, M. Vasilyev, and P. Kumar, *Frontiers in Optics 2010*, Rochester, NY, paper JTuB5.
[4] C. Schwob, P. F. Cohadon, C. Fabre, M. A. M. Marte, H. Ritsch, A. Gatti, and L. Lugiato, Appl. Phys. B **66**, 685 (1998).
[5] K. G. Köprulu and O. Aytür, Phys. Rev. A **60**, 4122 (1999).
[6] M. Vasilyev, M. Annamalai, N. Stelmakh, and P. Kumar, J. Mod. Opt. **57**, 1908 (2010).